Drawing Two Pairs[1]

A 5-card poker hand is called "two pairs" if it contains:

- two cards of one rank,
- two cards of a different rank,
- and 1 card of a third rank.

Find the probability of obtaining two pairs when a 5-card hand is drawn randomly from a standard 52-card poker deck.

Theoretical Solution Let's start with the denominator—that is, how many possible 5-card hands can we draw from a 52-card deck? Answer: $\binom{52}{5}$. Note, that this is the area of our sample space.

Now, we need to compute the portion of the sample space that corresponds to drawing two pairs. Just to review, a 52-card deck has 13 ranks and 4 suits ($4 \times 13 = 52$). We can choose a hand with two pairs by the following sequence of choices:

- Choose 2 ranks (out of 13) for the two pair. This can be done $\binom{13}{2}$ ways.
- Choose 2 suits (out of 4) for the smaller of the two chosen ranks. This can be done $\binom{4}{2}$ ways.
- Choose 2 suits (out of 4) for the larger of the two chosen ranks. This can be done $\binom{4}{2}$ ways.
- Choose the final card: There are 11 ranks remaining and the card can be of any suit. This can be done $\binom{11}{1}\binom{4}{1} = 11 \times 4 = 44$ ways.

Putting all of this together:

$$P(\text{two pair}) = \frac{\binom{13}{2}\binom{4}{2}\binom{4}{2}44}{\binom{52}{5}} = \frac{123,552}{2,598,960} = 0.047539$$

Solution via simulation See R script in folder.

[1]The problem and solution was written by Prof. Joesph Chang.